Physics Formula Sheet

Constants

Electron charge	$e = 1.6 \times 10^{-19}$ C
Speed of light	$c = 3 \times 10^{8}$ ms^{-1}
Plank's constant	$h = 6.63 \times 10^{-34}$ Js
Permittivity of free space	$\epsilon_0 = 8.85 \text{ x } 10^{-12}$ F m^{-1} (Farads/metre) or $C^2N^{-1}m^{-2}$
	$1/(4\pi\ \epsilon_0) = 9 \text{ x } 10^9$ Nm^{2}C^{-2}
Free space permeability	$\mu_0 = 4\pi \times 10^{-7}$ NA^{-2} or TmA^{-1}
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}$ JK $^{-1}$
Avogadro's number	$N_A = 6.023 \times 10^{23}$ / mole
Neutron mass	$m_n = 1.6 \times 10^{-27}$ kg
Earth surface gravity	9.8 m s^{-2} (sometimes approximated to 10)
Mass of earth	$M_E = 5.98 \text{ x } 10^{24}$ kg
Radius of earth	$R_E = 6.371 \text{ x } 10^6$ m
Electron charge	$e = -1.60 \text{ x } 10^{-19}$ C
Electron mass	$m_e = 9.11 \text{ x } 10^{-31}$ kg
Neutron mass	$1.675 \text{ x } 10^{-27}$ kg
Proton mass	$1.673 \text{ x } 10^{-27}$ kg
Universal gravitational constant	$G = 6.67 \text{ x } 10^{-11}$ N m^2 kg^{-2}
universal gas constant	R = 8.31 J/mol-K.

Average atmospheric pressure at sea level = 1 atmosphere = $1.013 \text{ x } 10^5$ N / m^2 .

Angular Momentum

Angular velocity ω (rad.s^{-1})
Usually in radians/s. This is a vector with direction given by the right hand rule.
Also revolutions/second (rev/s)
Or rotations/minute (rpm)

Moment of Inertia I (kg.m)
Consider a body made up of masses m_i each distance r_i from an axis of rotation.
Then $I = \sum m_i r_i$.
Examples:
Thin cylinder $I = M R^2$
Solid cylinder $I = (M R^2) /2$
Solid sphere $I = 2 (M R^2) /5$

Torque (N.m)
Torque = moment of inertia x angular acceleration
= distance_from_centre x perpendicular_force
Unit of angular acceleration is rad s^{-2}.

Rotational kinetic energy (J)
Rotational kinetic energy = $(I \omega^2)/2$

Angular momentum (kg.m.s^{-1})

Angular momentum $L = I\ \omega$
Angular momentum is conserved in the absence of a net torque.

Parallel axis theorem
$I = I_C + Mh^2$
Where I = moment of inertia of body, I_C = moment of inertia about central axis, M = mass of body, h = distance between the two axes.

Conservation and Motion

Newton's Laws
Every object in a state of uniform motion will remain in that state of motion unless an external force acts on it.
Force equals mass times acceleration. ($F = ma$)
For every action there is an equal and opposite reaction.

Terms:
s = distance
u = initial speed
v = final speed
a = acceleration
F = force
P = momentum
K = kinetic energy
k = spring constant
T = period
t = time
h = height above reference point such as ground level
m = mass
g = gravitational acceleration (9.8 m/s^2 at the surface of the earth).
G = gravitational constant

Linear Motion
$v = ds/dt = \Delta s/\Delta t$
$a = dv/dt = \Delta v/\Delta t$
momentum $P = mv = dE/dt$
kinetic energy $K = ½\ m\ v^2$
impulse = $\Delta P = F\ \Delta t$, where Δt is the duration of the applied force.
Work = W = force x distance = Fd
Potential energy = mgh
Power = W/t

For constant acceleration:
$v = u + at$
$s = ut + ½\ at^2$
$v^2 = u^2 + 2as$
$s = ½\ (u + v)\ t$

Conservation Laws

Total momentum is conserved.
Total energy is conserved.
Total energy = kinetic energy + potential energy.
In an elastic collision (e.g. incompressible hard balls) no energy is lost to heat.
In an inelastic collision (e.g. compressible soft balls) some energy is lost to heat.

A conservative force is a force with the property that the total work done in moving a particle between two points is independent of the path taken. Friction is non-conservative as energy is lost to heat. Gravity and an ideal spring are conservative.

Elastic and inelastic collisions
In a perfectly elastic collision there is no loss of kinetic energy. In an inelastic collision part of the kinetic energy is changed to some other form of energy such as heat in the collision. Momentum is conserved in both cases.

Circular Motion
$a = v^2/r = (4\pi^2 r)/T^2$
$v = (2 \pi r)/T$
angular velocity $\omega = d\theta/t$, where θ is angle in radians.
Centrifugal force (away from the centre) = $(m v^2)/r$
Centripetal force is towards the centre. This is required for circular motion and balances the centrifugal force.

Static Forces, Friction
Maximum or limiting Friction $F = \mu N$, where μ is the coefficient of friction.
This is the friction when a body starts to slip.
Until slippage the friction force is that needed to prevent slippage.
After slippage starts the kinetic or dynamic friction force remains at the limiting friction.

Simple Harmonic Motion
Hooke's Law: $F = -kx$, where x is the displacement of mass m from the unstretched position and k is the constant of elasticity.
Elastic potential energy = $\frac{1}{2} k x^2$
$x = \sin((2 \pi t)/T)$
$T = (2 \pi) \sqrt{(m/k)}$
$dx/dt = ((2 pi)/T) \cos((2 \pi t)/T)$
$d^2x/dt^2 = - ((2 pi)/T)^2 \sin((2 \pi t)/T)$

Pendulum
The period T of a pendulum length L is $(2 \pi) \sqrt{(L/g)}$.
This is independent of the mass suspended from the pendulum and the amplitude of the swing.

Mass
The mass of a body is its resistance to acceleration when a net force is applied.
The weight of a body is the gravitational force exerted on a body.
In common usage a body with mass m kg may be said to have a weight of m kg. This means that its weight is mg Newtons.
The center of mass is the average position of all the parts of the system, weighted according to their masses.

Gravity

Gravitational potential energy = mgh

This is relative to ground level. Unit = Joule.

Newton's law of universal gravitation: $F = (G\, m_1\, m_2)/r^2$

Gravitational field distance r from the centre of a planet mass M: $g = (GM)/r^2$

Gravitational potential energy of mass m = $U = -(GmM)/r$

Gravitational potential at a point is the work done in bringing a unit mass from infinity to that point without acceleration. Unit is J/kg.

For a satellite in orbit:

$$R^3/T^2 = (GM)/(4\,\pi^2)$$

Escape velocity v_e is the minimum speed needed for a free object to escape from the gravitational influence of a massive body.

$v_e = \sqrt{(2GM/r)}$, where M is mass of body to be escaped from, r = distance from centre of mass.

Planetary motion:

Kepler's first law: every planet's orbit is an ellipse with the Sun at a focus.

Kepler's second law: a line joining the Sun and a planet sweeps out equal areas in equal times. (This is equivalent to conservation of angular momentum.)

Kepler's third law: the square of a planet's orbital period is proportional to the cube of the semi-major axis of its elliptical orbit.

Electricity and Magnetism:

R = resistance

C = capacitance

E = electric field

P = power

I = current

F = force

E_P = potential energy

Q = charge

D = distance

B = magnetic field

R = radius

W = work done

L = length

v = velocity

ϕ = magnetic flux

A = area

V = IR

P = IV = W/t

E = V/d

F = EQ

F = BIL (force on a wire with current I in field B)

$F = QvB$ (for velocity at right angles to B)

$V = BvL$ (voltage induced in a wire moving through field B). (Faraday's Law)

$E_P = EQd = QV/2$

$I = Q/t$

$V = \Delta E/q$

$P = E/t$ = energy/time

$R_T = R_1 + R_2 + R_3$; resistors in series

$1/R_T = 1/R_1 + 1/R_2 + 1/R_3$; resistors in parallel

$B = (\mu_0 I)/(2 \pi r)$; magnetic field in Tesla, distance r from infinite wire with current I.

$\mu_0 = 4 \pi \times 10^{-7}$

$Q = CV$

$C = (\epsilon_0 \epsilon_r A)/d$ Capacitance between two plates area A, separation d.

$C_T = C_1 + C_2 + C_3$;capacitors in parallel

$1/C_T = 1/C_1 + 1/C_2 + 1/C_3$; resistors in series

Capacitor time constant: $\tau = R C$
Charging a capacitor: $V = V_0 (1 – e^{t/\tau})$
Discharging a capacitor: $V = V_0 e^{-t/\tau}$

$\varphi = BA$

EMF = electromotive force = $-L \Delta I / \Delta t = - \Delta\varphi / \Delta t$

Electromagnetic Fields

Electric field lines are drawn from a + charge to a – charge.

Magnetic field lines are drawn from a north magnetic pole to a south magnetic pole.

$v = f \lambda$
$f = 1/T$; T = period
$E = V/d$

Error Analysis

Absolute error = | measured – mean |
Relative error = | measured – mean |/mean
Percentage error = relative_error x 100

Standard error = standard_deviation / mean

Relative error for volume of a box = sum of relative errors for the sides.

Standard error for volume of a box = sum of standard errors for the sides.

Fluid Mechanics

Pressure
Density of water 1 g/cm^3 = 1000 kg/m^3

Specific gravity of a substance is its density relative to pure water.

Pressure p = |force| / area. It is a scalar.

Pressure unit Pascal Pa = N / m^2

Average atmospheric pressure at sea level = 1 atmosphere = 1.013 x 10^5 N / m^2 .

Pressure in a fluid density d at depth h due to the fluid is: p = dgh.
Atmospheric pressure is in addition to this.

A pressure gauge measures the difference between an unknown pressure and atmospheric pressure.

Archimedes Principle
The buoyant force on a submerged object equals the wight of fluid displaced.

Hydraulic Press or Lift
External pressure on a fluid is transmitted uniformly throughout the fluid. So in a closed liquid system a small piston of area A moving distance x can be used to push a large piston of area B through a distance y. Ax = By. So a small force can be used to lift a heavy object.

Fluid Flow
Rate of flow in a pipe = velocity x cross-sectional area.

Bernoulli's Equation (or principle or theorem)
Consider an incompressible fluid of density d with negligible viscosity or internal friction.
Let v be the fluid velocity at a point, h be height, and g be gravitational acceleration. Then:

$$p + d\,g\,h + d\,v^2/2 = \text{constant}$$

This is derived from conservation of energy. Pressure p here can be gauge or absolute.

At constant height: $p + d\,v^2/2 = \text{constant}$
This means that at a fixed height pressure decreases as velocity increases.

Work against surface tension
Work = surface_tension x increase_of_surface_area
A bubble has two surfaces: inside and outside.

Capillarity
Capillary action is the process of a liquid flowing in narrow spaces without the assistance of, or even in opposition to, external forces like gravity.
The rise of water in a thin tube inserted in water is caused by forces of attraction between the molecules of water and the glass walls and among the molecules of water themselves. The narrower the bore of the capillary tube, the higher the water rises.

Gases

Boyle's Law
At constant temperature in a gas PV is constant, where P is pressure and V is volume.

Absolute Temperature Scale
At a temperature of -273^o Celsius or 0^o Kelvin particles of a gas would lose all kinetic energy. This is the minimum possible temperature.

Charles's Law
At constant pressure V/T is constant, where T is the absolute temperature.

Molecular Energy
The average kinetic energy of a gas molecule = (3kT)/2 ,
Where k = Boltzmann's constant = 1.38 x 10^{-23} J/K.

Law of equipartition of energy
For any dynamical system in thermal equilibrium the total energy is distributed equally amongst all the degrees of freedom and the energy per molecule per degree of freedom is kT/2 where k is Boltzmann's constant and T is the absolute temperature.
A monomic gas molecule has 3 degrees of freedom. These are translational.
A diatomic gas molecule normally has 5 degrees of freedom. These are 3 translational, 2 rotational. An extra 2 vibrational degrees of freedom only occur at high temperature.

The Mole
Avogadro's number N = 6.023 x 10^{23} .
A mole of a substance contains N molecules.
A mole is defined for convenience. For example a mole of water has mass 18 grams because a water molecule has a mass of 18 atomic mass units.
1 atomic mass unit = 1.660 x 10^{-27} kg.

Ideal Gas Law
(PV)/T is constant.
OR
PV = nRT,
where n is the number of moles of gas and R = universal gas constant = 8.31 J/mol-K.

If P or V is doubled, then T doubles. If T doubles the average kinetic energy of a molecule doubles and so the average molecular speed increases by the factor $\sqrt{2}$.

Triple Point
The triple point is the temperature and pressure at which solid, liquid, and vapor phases of a particular substance coexist in equilibrium.

CP and CV of a gas
The molar specific heat of a gas at constant pressure (C_P) is the amount of heat required to raise the temperature of 1 mol of the gas by 1 °C at the constant pressure. Its value for monatomic ideal gas is 5R/2 and the value for diatomic ideal gas is 7R/2.
The molar specific heat of a gas at constant volume (C_V) is the amount of heat required to raise the temperature of 1 mol of the gas by 1 °C at the constant volume.
$C_P = C_V + R$
$C_V = dU/dT$, where U is the energy of a mole of gas.
Ratio of specific heats = $\gamma = C_P/C_V$.

Gauge pressure and absolute pressure
Gauge pressure is measured relative to atmospheric pressure and thus varies with height, location and conditions.
Absolute pressure is relative to 0. It equals gauge pressure plus atmospheric pressure.

Mean free path of a gas molecule
Average distance over which a moving particle substantially changes its direction or energy.
This can be calculated as: (volume of gas per molecule) /(average_cross-section x average_speed).

Standard temperature and pressure
0^o Celsius and 100 kPa.

Heat

Data:
m = mass
c = specific heat capacity
L = latent heat. This is heat of fusion (for melting) or heat of vaporisation.
Q = heat
T = temperature
$Q = m\,c\,\Delta T$
$Q = m\,L$

Specific heat capacity of ice:	2100 J kg^{-1} $^oC^{-1}$
Specific heat capacity of water:	4200 J kg^{-1} $^oC^{-1}$
Latent heat of fusion of water:	330000 J kg^{-1}
Latent heat of vaporisation of water:	2300000 J kg^{-1}

1 kilocalorie = 1000 calories = 4186 joules = heat needed to heat 1 kg of water by 1^o Celcius.

Temperature
The temperature of a body is a measure of the average kinetic energy of its particles.

Water freezes at 0^o Celsius and boils at 100^o Celsius.

Increasing pressure increases boiling point.

Thermal conductivity K
$K = (Q\,d)/(A\,\Delta T)$,

where Q = heat transferred, d = distance between isothermal or constant temperature planes, A = surface area, and ΔT = temperature difference.

Thermodynamics
First law:
The internal energy of a body may be increased by adding heat Q or by doing work W.

Increase in internal energy = Q + W

For an adiabatic process no heat is added so that Q = 0.

A heat engine converts internal energy into mechanical energy. Examples are the human body, the earth's atmosphere and motors.

Second law:
It is impossible to build a continuously operating engine that takes heat from a source and performs an exactly equivalent amount of work.
There is always some heat loss.

Engine efficiency = work_output/heat_input = $1 - T_2 / T_1$,
where heat is absorbed at T_1 and given out at T_2.

Heat engine
Consider heat Q_H from source at temperature T_H which is used to do work W with lost heat Q_L going to sink at temperature T_L.

$W = Q_H - Q_L$

$Q_H/Q_L = T_H/T_L$

Efficiency for ideal or Carnot engine = $W/Q_H = 1-Q_L/Q_H = 1- T_L/T_H$

Light and Matter Waves

Light and matter

Photelectric effect	$E_{K\_max} = hf - W$
Photon energy	$E = hf = hc/\lambda$
Photon momentum	$p = h/\lambda$
De Broglie wavelength	$\lambda = h/p$
Heisenberg's uncertainty principle	$\Delta p_x \Delta x >= h/(4\pi)$
Wave number k	$(2\pi)/\lambda$
Angular frequency ω	$2\pi f$

Diffraction grating:
n = order of diffraction line.
d = grating spacing
λ = wavelength
θ = angle of diffraction
$\sin(\theta) = (n\lambda)/d$

Nodes and antinodes:
A node is a position of minimum displacement.
An antinode is a position of maximum displacement.

Sound:
$v = f \lambda$
An open pipe length L has an antinode at each end. $L = n \lambda$ where n is a positive integer.
The first harmonic is the lowest frequency of sound in the pipe.
For a pipe with one end closed and one open, the closed end is a node. $L = (2n - 1) \lambda$.

Progressive and stationary waves
A progressive or travelling wave is a wave that travels continuously in a medium in the same direction without a change in amplitude, while a standing wave is a wave in which the position of the maximum and minima do not travel, but remain in place.
General travelling wave in x direction: $A \cos(kx - \omega t)$
General standing wave: $A \cos(kx) \cos(\omega t)$

Doppler effect for sound:
The *Doppler effect* or *Doppler shift* is the change in frequency of a wave in relation to an observer who is moving relative to the source. The frequency increases if the observer approaches the source.
Let f_o = observer frequency, f_s = source frequency, v = speed of sound, v_o = observer velocity and v_s = source velocity, then:

$f_o = f_s (v + v_o)/(v + v_s)$

Optics

d = distance
o = object
i = image
f = focal length
m = magnification
h = height
n = refractive index
θ_1 = angle of incidence (between normal and incident ray)
θ_2 = angle of refraction (between normal and exit ray)
v = velocity of light in a medium

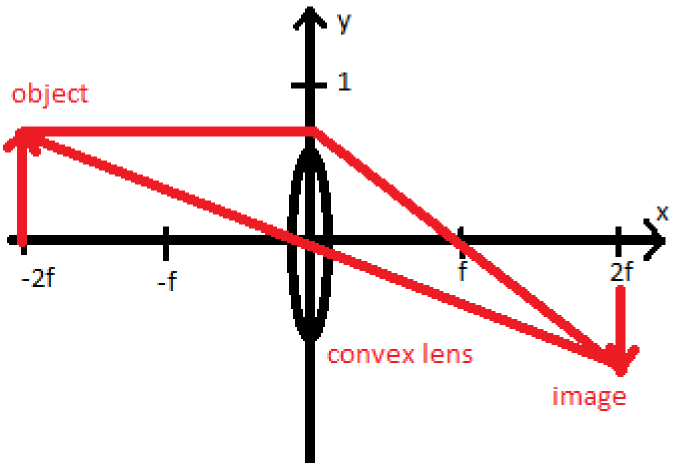
Mirror:
Angle of incidence = angle of reflection.

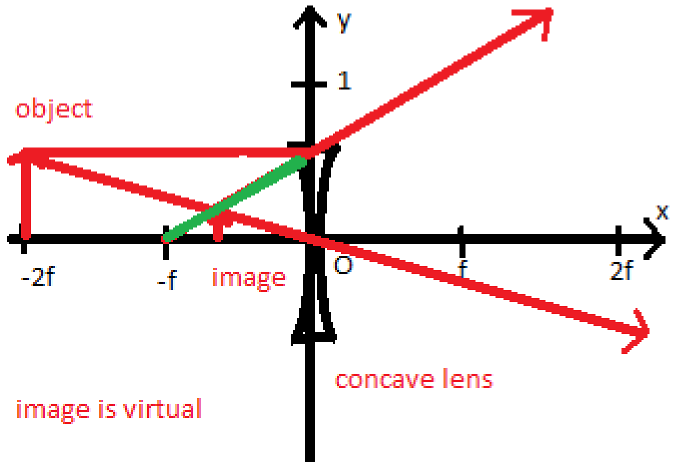
Snell's Law for refraction:
$n_1 \sin \theta_1 = n_2 \sin \theta_2$

Total internal reflection:
$\theta_2 = 90^o$.
So: $\sin \theta_1 = n_2 / n_1$, where $n_2 < n_1$.

Lens equations:
$1/f = 1/d_o + 1/d_i$
$m = d_i / d_o = h_i / h_o$
A negative magnification indicates inversion of the image compared to the object.
For a real object or image, d_o and d_i are positive.

For a real image light passes through or emanates from points on the image.
For a virtual object, d_o and d_i are negative.
For a virtual object light appears to pass through or emanate from points on the object.
f is positive for a convex lens or mirror and negative for a concave lens or mirror.
A convex lens is wider at the centre.
For a convex lens, parallel light from the left is focussed at the focal plane on the right of the lens.
For a concave lens, parallel light from the left apparently diverges on the right of the lens from the focal plane on the left of the lens.
For a convex mirror facing left, parallel light from the left apparently diverges on the left of the mirror from the focal plane on the right of the mirror.
For a concave mirror facing left, parallel light from the left is focussed at the focal plane on the left of the mirror.
It helps to sketch a diagram of the image formation.





Refractive index:
$n_1 / n_2 = v_2 / v_1 = \lambda_2 / \lambda_1$

Relativity:

Relativistic mass $m = m_0 \gamma$ (γ always >1)

Total energy as seen by observer $E_{total} = E_k + E_{rest} = mc^2$

Time dilation $t = t_0 \gamma$ t_0 = proper time of observed. t = time for observer.

Length contraction $L = L_0 / \gamma$ L_0 = proper length of observed. L = length for observer.

Stress and strain:

Stress $\sigma = F/A$

Strain $\varepsilon = (\Delta L)/L$

Young's modulus E = stress/strain

Unit Conversions

Angles

The degrees of arc in a circle = 360.
180 degrees = π radians
1 degree of arc = 60 minutes
1 minute of arc = 60 seconds

Distance

1 meter = 1 metre = 1 m = 100 centimetres = 100 cm

1 inch = 2.54 cm

1 foot = 12 inches

1 yard = 3 feet

1 mile = 1760 yards

1 nautical mile = 1 minute of latitude = 1852 metres

Area

1 hectare = 1 ha = 10000 m^2

1 acre = 4840 square yards

Volume

1 m^3 = 1000L
1L = 1 litre = 1000cc = 1000 cubic centimetres

1 pint = 0.56826 litres

1 quart = 2 pints

1 gallon = 4 quarts

Information

1 bit = 0 or 1

1 byte = 8 bits

1 kilobyte = 2^{10} bytes = 1024 bytes = 1KB

1 megabyte = 2^{20} bytes = 1024 kilobytes = 1MB

1 gigabyte = 2^{30} bytes = 1024 megabytes = 1GB

1 terabyte = 2^{40} bytes = 1024 gigabytes = 1TB

Prefixes

p = pico = 10^{-12}

n = nano = 10^{-9}

µ = micro = 10^{-6}

m = milli = 10^{-3}

centi = 10^{-2}

deci = 10^{-1}

deca = 10

k = kilo = 10^{3} = 1 thousand

M = mega = 10^{6} = 1 million

G = giga = 10^{9} =1 billion

Weight

t = tonne = 10^{3} kg

16 ounces = 1 pound = 1 lb = 0.4536 kg

1 ton = 2240 lb

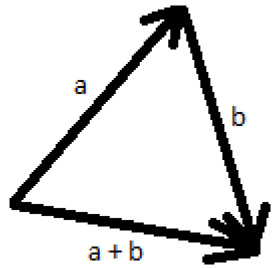
Temperature:

A^{o} C = (B^{o} F -32) x (100/180)

Vectors:

A vector is a quantity with magnitude (or length) and direction.
A scalar is a quantity with magnitude but no direction.

Addition of vectors:



Dot product of two vectors with angle θ between them:

a.b = a b cos(θ)

This is also called the scalar resolute of **a** on **b**.
The dot product is a scalar.
Vectors a and b are parallel if **a.b** = a b
Vectors a and b are perpendicular if **a.b** = 0
If **a** has components (a_1, a_2) and **b** has components (b_1, b_2) then **a.b** = $a_1b_1 + a_2b_2$

The modulus of a vector **b** is the scalar length of the vector |**b**| or b.

Projection of vector **a** on **b** = (**a.b**)/|**b**|^2 **b**
This is also called the vector resolute or resolute of **a** in the direction **b**.

Linear dependence

A set of vectors are linearly dependent if at least one of them can be written as a linear combination of the others.
One way of proving linear dependence is to make a matrix from the vectors. If the determinant is zero, the vectors are linearly dependent.

Cross product

Cross product of two vectors with angle θ between them:

a x b = a b sin(θ) **n**

where **n** is a unit vector perpendicular to **a** and **b**.

The direction of **n** is given by the righthand rule: If **a** is in the direction of your second finger, **b** is in the direction of your third finger, then **n** is in the direction of your thumb.

3-D vectors

Let **i**, **j** and **k** be unit vectors in the directions of the x, y and z axes respectively.

These can be used to specify a general vector. E.g.: **r** = x**i** + y**j** – z**k**

|**r**| = r = $\sqrt{(x^2 + y^2 + z^2)}$

d**r**/dt = dx/dt **i** + dy/dt **j** + dz/dt **k**

j is normal to the xz plane. The angle between a vector and this plane = 90° – angle between **j** and the vector.

Mechanics:

m = mass, **v** = velocity, **a** = acceleration.

Momentum **p** = m**v**

Force **F** = m**a**

Friction: F <= μN

(here F is the friction force magnitude, N is the normal force magnitude, and μ is the coefficient of friction.)