

Geometry: (Level 1)

Map bearings, time difference and nautical miles

True bearing is measured clockwise in degrees from North.

For calculating time difference using longitude:

$15^\circ = 1$ hour time difference.

1 nautical mile = 1 minute of latitude = 1 minute of longitude at equator.

At latitude θ° , 1 minute of longitude is $\cos(\theta)$ nautical miles.

1 degree = 60 minutes. 1 minute = 60 seconds.

Conventions:

Two dashes across each of a pair of lines indicate they are parallel.

A single dash across each of a pair of line segments indicate that they have equal length.

A small square drawn in the corner of a figure indicates a right angle.

Angle types:

Reflex $>180^\circ, <360^\circ$

Acute $>0^\circ, <90^\circ$

Obtuse $>90^\circ, <180^\circ$

Right 90°

Straight 180°

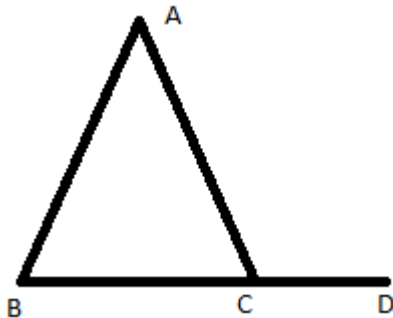
Complementary angles: two angles whose sum is 90°

Supplementary angles: two angles whose sum is 180°

Opposite angles: Formed when two lines intersect at a point.

Interior angles: A triangle has 3 interior angles.

Exterior angles: In the diagram below angle ACD is exterior to triangle ABC. It equals the sum of angles CBA and BAC.



2D shapes:

Triangle (3 sides)

Quadrilateral (4 sides)

Trapezium (quadrilateral with 1 pair of opposite sides parallel)

Parallelogram (quadrilateral with 2 pairs of sides parallel)

Rectangle (quadrilateral with each angle 90° .)

Rhombus (quadrilateral with 4 sides equal)

Square (quadrilateral with each side equal and each angle 90° .)

Kite (quadrilateral in which 2 pairs of adjacent sides are equal)

Pentagon (5 sides)

Hexagon (6 sides)

Heptagon (7 sides)

Octagon (8 sides)

Notes:

A vertex is a point where two lines meet.

A shape such as a hexagon is regular if all the sides and interior angles are equal.

A right triangle includes a right angle.

Square, rectangle and rhombus are all parallelograms.

Kite and trapezium are not parallelograms.

Square is a rectangle and a rhombus.

Parallelogram is a trapezium.

The diagonals of a parallelogram bisect each other.

The diagonals of a rhombus and a square meet at right angles.

Within a parallelogram adjacent angles add to 180° and opposite angles are equal.

Triangles: Sum of the angles = 180° .

A right triangle includes a right angle.

An obtuse triangle includes an obtuse angle.

An isosceles triangle includes two equal angles and has two equal sides.

An equilateral triangle has three equal sides and three 60° angles.

A scalene triangle has three different angles.

The angles opposite to equal sides of a triangle are equal.

The incircle of a triangle is the largest circle that can be drawn within a triangle. Its centre is the intersection of the angle bisectors from the vertices.

The centroid of a triangle is the intersection of the medians. A median is the line from a vertex to the mid-point of the opposite side. The centroid is also called the centre of gravity of a triangle. It is the mean coordinate of the vertices.

Circles: A chord is a straight line between two points on the perimeter.

An arc is the part of the perimeter between two points on the perimeter.

A sector is the area between an arc and the centre.

A segment is the smaller area between a chord and the circle.

The tangent to a circle is perpendicular to the radius at the point of contact.

Tangents to a circle from an external point are equal.

The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.

When circles touch, the line through the centres passes through the point of contact.

Circumscribed circle of a polygon:

This passes through all vertices of the polygon.

The centre is the circumcenter.

A triangle and rectangle can always be circumscribed.

A triangle circumcenter is at the intersection of the perpendicular bisectors of the sides.

The orthocenter of a triangle is the point of intersection of the altitudes. The altitude is the line from a vertex perpendicular to the opposite side. It is not the same as the circumcenter.

Quadrilaterals: Sum of the internal angles = 360° .

Cyclic quadrilaterals:

A quadrilateral is cyclic if a circle can be drawn through its vertices.

For a cyclic quadrilateral:

sum of opposite angles = 180° .

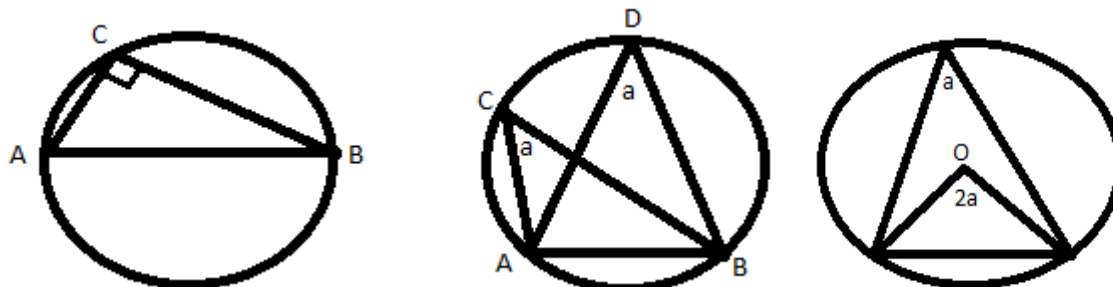
if the opposite angles in a quadrilateral are supplementary then the

quadrilateral is cyclic.

a diameter subtends a right angle. (see left diagram)

a single chord subtends a constant angle. (see middle diagram)

a chord subtends an angle at the centre which is twice the angle subtended at a point on the circle. (see right diagram). Note that this is true even if the angle of size $2a$ shown at right below is reflex.



Similar and congruent triangles

Two triangles are similar if: two angles are the same (AA), or three sides are in proportion.

Two triangles are congruent if:

three sides are the same (SSS),

or two sides are the same and the included angle is the same (SAS),

or two angles and the included side are the same (ASA),

or two angles and any corresponding side pair are the same (AAS),

or for two right triangles if the hypotenuse and one other side or leg are equal in the other triangle (HL).

Congruent triangles can be made to cover each other exactly using some or all of rotation, translation and reflection. For direct congruence this involves rotation and translation. For opposite congruence a reflection is also needed.

CPCT is short for: corresponding parts of congruent triangles.

In the abbreviation ASA, A means two corresponding angles are the same, S means two corresponding sides are the same. Order is significant.

Isosceles triangle: has two equal sides. The angles opposite the equal sides are equal.

Equilateral triangle: has three equal sides. The angles are all 60° .

Pythagoras theorem $c^2 = a^2 + b^2$; a, b and c are the sides of a right angled triangle.

If a, b and c are integers they are called a Pythagorean triple. Examples are: 3, 4, 5; 5, 12, 13; 7, 24, 25.

Triangle sides: If AB is the longest side of triangle ABC then $AB < BC + CA$.

Sum of interior angles of polygon with n sides

$$S = (n - 2) \times 180^\circ$$

The interior angle of a polygon with n equal sides is: $((n - 2)/n) \times 180^\circ$

3D shapes or solids:

Triangular prism. (base is a triangle)

Rectangular prism. (i.e. base is rectangle)

Square prism

Square based pyramid.

Triangular pyramid

Rectangular pyramid

Cube

Note: diagonal of a unit cube = $\sqrt{3}$ units.

Box or cuboid (6 faces, 12 edges, 8 vertices)

Sphere

Cylinder

Cone

Frustum (truncated cone)

Equidistant points:

A point equidistant from two given points lies on the perpendicular bisector of the line segment joining the two points.

A point equidistant from two intersecting lines lies on the bisectors of the angles joined by the two lines.