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Calculus:

Differentiation from first principles:

$$f'(x) = \text{limit as } h \text{ approaches } 0: (f(x+h) - f(x))/h$$

$$d(f(x))/dx = f'(x) = y' = dy/dx = (d/dx)(y)$$

Differentiation:

Function	Derivative	
x^n	nx^{n-1}	
$(ax + b)^n$	$an(ax + b)^{n-1}$	
uv	$u dv/dx + v du/dx$	Product rule
u/v	$(v du/dx - u dv/dx)/v^2$	Quotient rule
$f(u)$	$f'(u) du/dx$	
$e^{f(x)}$	$f'(x) e^{f(x)}$	
e^{ax}	$a e^{ax}$	
$\log_e f(x) = \ln f(x)$	$f'(x) / f(x)$	
$\log_e x = \ln(x)$	$1/x$	
$\sin(f(x))$	$f'(x) \cos f(x)$	
$\sin(ax)$	$a \cos(ax)$	(x is in radians)
$\cos(f(x))$	$-f'(x) \sin f(x)$	
$\cos(ax)$	$-a \sin(ax)$	
$\tan(f(x))$	$f'(x) \sec^2 f(x)$	
$\tan(ax)$	$a \sec^2(ax)$	
$\sec(x)$	$\sec(x) \tan(x)$	
$\csc(x) = \operatorname{cosec}(x)$	$-\operatorname{cosec}(x) \cot(x)$	
$\cot(x)$	$-\operatorname{cosec}^2(x)$	
$\sin^{-1}(x/a) = \arcsin(x/a)$	$(a^2 - x^2)^{-1/2}$	$a > 0$
$\cos^{-1}(x/a) = \arccos(x/a)$	$-(a^2 - x^2)^{-1/2}$	$a > 0$
$\tan^{-1}(x/a) = \arctan(x/a)$	$a(a^2 + x^2)^{-1/2}$	
$f(g(x))$	$f'(g(x)) g'(x)$	
$\ln(f(x))$	$f'(x)/f(x)$	

Chain rule:

$$dy/dx = dy/du \cdot du/dx$$

Product rule:

$$y = f(x) g(x)$$

$$dy/dx = f(x) g'(x) + f'(x) g(x)$$

Quotient rule:

$$y = f(x) / g(x)$$

$$dy/dx = (f'(x) g(x) - f(x) g'(x)) / (g(x))^2$$

Parametric function:

$$d^2y/dx^2 = d(dy/dx)/dt \cdot dt/dx$$

Integration:

Fundamental theorem of calculus:

Area under graph $y = f(x)$ between $x = a$ and $x = b$ is:

$$\begin{aligned} \int_a^b f(x) dx &= [F(x)]_a^b, \text{ where } F'(x) = f(x) \\ &= F(b) - F(a) \end{aligned}$$

Note: $\int_a^b f(x) dx = -\int_b^a f(x) dx$

Example:

$$\int e^x dx = e^x + C$$

Function	Integral	
a	$ax + C$	
x^n	$(1/(n+1)) x^{n+1} + C$	$n \neq -1$
$(ax + b)^n$	$(ax + b)^{n+1} / (a(n+1)) + C$	C is a constant, $n \neq -1$
e^{ax+b}	$(1/a) e^{ax+b} + C$	
$f'(x)/f(x)$	$\log_e(f(x)) + C$	
$\sin(ax + b)$	$-(1/a) \cos(ax + b) + C$	
$\cos(ax + b)$	$(1/a) \sin(ax + b) + C$	
$\sec^2(ax + b)$	$(1/a) \tan(ax + b) + C$	
$e^x (f(x) + f'(x))$	$e^x f(x) + C$	
$(a^2 - x^2)^{-1/2}$	$\sin^{-1}(x/a) + C$	$a > 0$
$-(a^2 - x^2)^{-1/2}$	$\cos^{-1}(x/a) + C$	$a > 0$
$(a^2 + x^2)^{-1/2}$	$(1/a) \tan^{-1}(x/a) + C$	
$1/x$	$\log_e(x) + C$	$x > 0$
$(ax + b)^{-1}$	$(1/a) \log_e(ax + b) + C$	$x > 0$

Stationary points:

At a stationary point $f'(x) = 0$.

Type	y' before, at and after	$y'' = d^2y/dx^2$
Local minimum	$-, 0, +$	$+$
Local maximum	$+, 0, -$	$-$
Point of inflexion	$+, 0, +$ OR $-, 0, -$	0

Average rate of change:

Over interval a to b , average rate of change = $(f(b) - f(a))/(b - a)$

Volume of solid of revolution about x-axis:

$$\pi \int_a^b y^2 dx$$

Integration by parts:

$$\int u dv = uv - \int v du$$

Approximations:

Linear approximation:

$$f(x+h) \approx f(x) + h f'(x)$$

Trapezoidal rule (one application) or mid-point rule

$$\int_a^b f(x) dx \approx ((b-a)/2) (f(a) + f(b))$$

Simpson's rule (one application)

$$\int_a^b f(x) dx \approx ((b-a)/6) (f(a) + 4f((a+b)/2) + f(b))$$

Simpson's Rule:

$$\text{Area } A \approx (h/3) (d_f + 4d_m + d_l)$$

h = distance between measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

$$\text{Volume } V \approx (h/3) (A_L + 4A_M + A_R)$$

A_L = left area

A_M = middle area

A_R = right area

Using left rectangles or right rectangles for integral approximation over an interval:

For left rectangles the rectangles are under the curve for an increasing function.

For right rectangles the rectangles are above the curve for an increasing function.

Euler's method:

If $dy/dx = f(x)$, $x_0 = a$ and $y_0 = b$, then:

$$x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + h f(x_n)$$

Arc length along $f(x)$:

$$\int_a^b \sqrt{1 + (f'(x))^2} dx$$

Or in parametric form:

$$\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} dt$$