Calculus:

Differentiation from first principles:
f `(x) = limit as h approaches 0: (f(x + h) – f(x))/h
d(f(x))/dx = f `(x) = y` = dy/dx = (d/dx)(y)

Differentiation:

Function	**Derivative**	
x^n	nx^{n-1}	
$(ax + b)^n$	$an\ (ax + b)^{n-1}$	
uv	u dv/dx + v du/dx	Product rule
u/v	$(v\ du/dx – u\ dv/dx)/v^2$	Quotient rule
f(u)	f `(u) du/dx	
$e^{f(x)}$	f `(x) $e^{f(x}$)	
e^{ax}	a e^{ax}	
$\log_e f(x) = \ln f(x)$	f `(x) / f(x)	
$\log_e x = \ln(x)$	1/x	
sin(f(x))	f `(x) cos f(x)	
sin(ax)	a cos(ax)	(x is in radians)
cos(f(x))	-f `(x) sin f(x)	
cos(ax)	-a sin(ax)	
tan(f(x))	f`(x) $\sec^2$ f(x)	
tan(ax)	a $\sec^2(ax)$	
sec(x)	sec(x) tan(x)	
cosec(x)	-cosec(x) cot(x)	
cot(x)	$-\text{cosec}^2(x)$	
$\sin^{-1}$ (x/a) = arcsin(x/a)	$(a^2 – x^2)^{-1/2}$	a > 0
$\cos^{-1}$ (x/a) = arccos(x/a)	$-(a^2 – x^2)^{-1/2}$	a > 0
$\tan^{-1}$ (x/a) = arctan(x/a)	$a\ (a^2 + x^2)^{-1/2}$	
f(g(x))	f `(g(x)) g`(x)	
ln(f(x))	f `(x)/f(x)	

Chain rule:
dy/dx = dy/du du/dx

Product rule:
y = f(x) g(x)
dy/dx = f(x) g`(x) + f `(x) g(x)

Quotient rule:
y = f(x) / g(x)
dy/dx = (f `(x) g(x) - f(x) g`(x))/$(g(x)^2$

Parametric function:
D2y/dx2 = d(dy/dx)/dt dt/dx

Integration:
Fundamental theorem of calculus:
Area under graph y = f(x) between x = a and x = b is:

$\int_a^b f(x)\,dx$
$= [F(x)]_a^b$, where $F`(x) = f(x)$
$= F(b) – F(a)$
Note: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Example:
$\int e^x\,dx \qquad = e^x + C$

Function	**Integral**	
a	$ax + C$	
x^n	$(1/(n+1))\,x^{n+1} + C$	$n \neq -1$
$(ax + b)^n$	$(ax + b)^{n+1} /(a(n+1)) + C$	C is a constant, $n \neq -1$
e^{ax+b}	$(1/a)\,e^{ax+b} + C$	
$f\,`(x)/f(x)$	$\log_e(\,\vert f(x)\vert) + C$	
$\sin(ax + b)$	$-(1/a)\cos(ax + b) + C$	
$\cos(ax + b)$	$(1/a)\sin(ax + b) + C$	
$\sec^2(ax + b)$	$(1/a)\tan(ax + b) + C$	
$e^x\,(f(x) + f\,`(x))$	$e^x f(x) + C$	
$(a^2 – x^2)^{-1/2}$	$\sin^{-1}(x/a) + C$	$a > 0$
$-(a^2 – x^2)^{-1/2}$	$\cos^{-1}(x/a) + C$	$a > 0$
$(a^2 + x^2)^{-1/2}$	$(1/a)\tan^{-1}(x/a) + C$	
$1/x$	$\log_e(\,\vert x\vert\,) + C$	$x > 0$
$(ax + b)^{-1}$	$(1/a)\log_e(\,\vert ax + b\vert\,) + C$	$x > 0$

Stationary points:
At a stationary point $f\,`(x) = 0$.

Type	y` before, at and after	$y`` = d^2y/dx^2$
Local minimum	-, 0, +	+
Local maximum	+, 0, -	-
Point of inflexion	+, 0, + OR - ,0, -	0

Average rate of change:
Over interval a to b, average rate of change =
$(f(b) – f(a))/(b - a)$

Volume of solid of revolution about x-axis:
$\pi \int_a^b y^2\,dx$

Integration by parts:
$\int u\,dv = uv - \int v\,du$

Approximations:
Linear approximation:
$f(x + h) \sim= f(x) + h\,f\,`(x)$

Trapezoidal rule (one application) or mid-point rule
$\int_a^b f(x)\,dx \qquad \sim= ((b - a)/2)\ (f(a) + f(b))$

Simpson's rule (one application)

$\int_a^b$ f(x) dx ~= ((b - a)/6) (f(a) + 4f((a + b)/2) + f(b))

Simpson's Rule:

Area A ~= (h/3) (d_f + 4d_m + d_l)

h = distance between measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

Volume V ~= (h/3) (A_L + 4A_M + A_R)

A_L = left area

A_M = middle area

A_R = right area

Using left rectangles or right rectangles for integral approximation over an interval:

For left rectangles the rectangles are under the curve for an increasing function.

For right rectangles the rectangles are above the curve for an increasing function.

Euler's method:

If dy/dx = f(x), x_0 = a and y_0 = b, then:

$x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h\, f(x_n)$

Arc length along f(x):

$\int_a^b \sqrt{(1 + (f`(x))^2)}\, dx$

Or in parametric form:

$\int_a^b \sqrt{((x`(t))^2 + (y`(t))^2)}\, dt$