Algebra:

Expansions:

$(a + b)^2 = a^2 + 2ab + b^2$

$a^2 – b^2 = (a - b)(a + b)$ (difference of squares)

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 – b^3 = (a - b)(a^2 + ab + b^2)$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$x^3 + y^3 + z^3 – 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Modulus:

If $x > 0$, $|x| = |-x| = x$

If $x < 0$, $|x| = |-x| = -x$

Remainder theorem: If a polynomial f(x) is divided by (x - a) the remainder is f(a).

Polynomial example:

Let $y = f(x) = ax^2 + bx + c$.

This is a second order polynomial.

The coefficients are a, b and c.

The second term of the polynomial is bx.

Factor theorem: A polynomial f(x) has a factor (x - a) if and only if f(a) = 0.

Binomial theorem: k^{th} term of $(a + b)^n$ is $C_k^n a^{n-k} b^k$ for k = 0, 1,.., n.

For each value of n, the coefficients are given by row n of Pascal's triangle:

Row 0				1			
Row 1			1		1		
Row 2		1		2		1	
Row 3	1		3		3		1

Etc.

For example: $(1 + x)^3 = 1 + 3x + 3x^2 + x^3$

Simultaneous equations.

Solve by separating out an individual variable to obtain simpler equation(s). Solve these and then substitute back to find the full solution.

e.g.

Solve: $x + y = 3$ (1)

and $2x - 3y = 1$ (2)

So: $y = -x + 3$

and $y = (2x - 1)/3$

So: $-3x + 9 = 2x - 1$

$5x = 10$

$x = 2$

Substitute into equation (1) gives: $2 + y = 3$

So: $y = 1$

So solution is: $x = 2$, $y = 1$.

Alternative approach:
Add 3 times equation (1) to equation (2) to eliminate the y term.

$3x + 3y + 2x - 3y = 9 + 1$

$5x = 10$

$x = 2$

Substitute into equation (1) gives: $2 + y = 3$
So: $y = 1$
So solution is: $x = 2$, $y = 1$.

Areas and volumes:

Area:

Triangle: (1/2) base x height

Triangle with sides b, c and included angle A: $b\, c \sin(A)/2$

Triangle: Heron's formula for area of triangle with sides a, b, c.

$(s\,(s - a)(s - b)(s - c))^{1/2}$, where $s = (a + b + c)/2$

Circle: $\pi\, r^2$

Sector of a circle with angle x degrees: $(\pi\, r^2\, x)/360$
Sector of a circle with angle x radians: $(r^2\, x)/2$

Annulus with outer radius R, inner radius r: $\pi\,(R^2 - r^2)$

Surface area of sphere: $4\,\pi\, r^2$

Surface area of closed hemisphere: $2\,\pi\, r^2 + \pi\, r^2 = 3\,\pi\, r^2$

Surface area of closed cylinder with radius r, height h: $2\,\pi\, r^2 + 2\,\pi\, r\, h$

Curved surface area of a cylinder: $2\,\pi\, r\, h$

Parallelogram with opposite parallel sides a, and perpendicular height h: ah

Trapezium with parallel sides a and b, and perpendicular height h: $h\,(a + b)/2$

Area of a triangle given vertex coordinates: $(1/2)\,|\,a_x(b_y - c_y) + b_x(c_y - a_y) + c_x(a_y - b_y)|$

Curved surface area of a cone: $\pi\, r\, L$, where L is the length of the cone side.

Rhombus with diagonals a, b: $(ab)/2$

Volume:

Sphere: $(4\,\pi\, r^3)/3$

Cone height h: $(\pi\, r^2\, h)/3$

Prism or cylinder: area_of_base x height

Cylinder $\pi r^2 h$

Pyramid or cone: (area_of_base x height)/3 = Ah/3

Scale factors:
Increasing scale of an object by k increases area by k^2 and volume by k^3.

Distance Speed Time

Distance_travelled = speed x time

Average_speed = total_distance_travelled / total_time

Stopping_distance = reaction-time_distance + braking_distance

Speed v = dx/dt

x measures position or displacement from the origin.
If direction changes then the total distance travelled can be different from the net displacement.

Acceleration
$a = d^2x/dt^2 = dv/dt = v\, dv/dx = d(v^2/2)/dx$

Constant acceleration a:
Initial speed u, final speed v, distance s, time t.
$v = u + at$; $s = ut + ½ at^2$; $v^2 = u^2 + 2as$; $s = ½(u + v) t$

Simple harmonic motion
$x = b + a \cos(nt + c)$; a, b and c are constants.

$d^2x/dt^2 = -n^2 (x – b)$

Interest and finance

Interest

R = annual interest rate as a percentage
N = payments/year
P = principal or initial amount
T = time in years
I = interest paid
r = interest rate per period expressed as a decimal = R/(100 N)
n = number of compounding periods = NT

Simple interest:	I = PRT/100 = P r n
Compound interest:	$I = P (1 + R/(100N))^{NT} - P = P(1 + r)^n - P$
Final amount:	A = P + I
Number of compounding periods:	NT

Effective rate of interest for a compound interest loan or investment:
$r_{effective} = 100 ((1 + r/(100N))^N - 1)$ %

Present Value PV and future value FV
$PV = FV/(1 + r)^n$
$FV = PV (1 + r)^n$

Depreciation:
S = salvage value of asset after n periods
V_0 = initial value of asset
D = depreciation per period
n = number of periods
r = depreciation rate per period expressed as a decimal
Straight line method of depreciation:
$S = V_0 – D n$
Declining balance method of depreciation:
$S = V_0 (1 – r)^n$

Rule of 72
If the annual compound interest rate is r% then the time to double an investment is approximately 72/r.

Hire purchase:
$r_f = (100 I M)/(PN)$ = flat rate of interest paid for hire purchase
I = total interest paid = repayments – principal repayments
P = principal – deposit
M = number of repayments/year
N = total number of repayments or periods
$r_e = r_f (2N)/(2N + 1)$ = effective rate of interest

First-order linear recurrence relation
$u_0 = a$, $u_{n+1} = b\, u_n + c$

TVM
TVM = total value of money. A TVM solver calculator is used to calculate this.

Lines

Straight line equation (slope intercept form): $y = mx + c$; m = gradient, c = y-intercept.

The slope of a perpendicular line is -1/m.

Gradient or slope = $m = (y_2 – y_1)/(x_2 – x_1)$ = vertical_change/horizontal_change

For x-intercept: y = 0, so x = -c/m

For y-intercept: x = 0, so y = c

Distance between two points:
$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

Perpendicular distance of point (x_1, y_1) from line $ax + by + c = 0$:
$D = |ax_1 + by_1 + c| / \sqrt{(a^2 + b^2)}$

Two lines are parallel if their gradients are the same.
If two straight lines are parallel they never intersect unless they are identical.
Two lines are perpendicular if the product of their gradients is -1.

Linear equation in two variables:
$ax + by + c = 0$
a, b and c are real numbers, and x and y are variables. a and b are called coefficients and c is the constant term. This equation can be rewritten in the simpler straight line equation form as above.

Logs exponentials and powers:

Logarithms:

If $a = 10^x$ then: $\log_{10}(a) = x$. The base here is10.
$a = 10^{\log 10(a)}$
$\log_a(1/2) = -\log_a(2)$
If $a = 1000 = 10 \times 10 \times 10 = 10^3$, $\log_{10}(a) = 3$
In some syllabuses the default base is 10 so that $\log(a) = \log_{10}(a)$.

If $x = e^y$ then: $\log_e(x) = \log_e x = \ln(x) = \ln x$ = natural log of $x = y$

Change of base:
$\log_a(b) = \log_e(b)/\log_e(a)$
or more generally:
$\log_a(b) = \log_c(b)/\log_c(a)$

If $a = b\ c$, then: $\log(a) = \log(b) + \log(c)$

If $a = b/c$, then: $\log(a) = \log(b) - \log(c)$

If $a = b^c$, then: $\log(a) = c \log(b)$

If $y = a^x$ then $\log_a(y) = x$
$\log_a(y) = 0 = \ln(1)$
$\log_a(a) = 1 = \ln(e)$

Powers:

$1 = 0! = 0^0 = 1^0 = x^0$ (The value of 0^0 is debatable and can be taken as undefined or 1.)

$(x^m)/(x^n) = x^{m-n}$

$x^m x^n = x^{m+n}$

$(x^m)^n = x^{mn}$

$x^{1/2}$ = sqrt(x)

$x^{-1} = 1/x$

$(x/y)^n = x^n/y^n$

Mental Arithmetic

Multiplying by 5:
E.g: 5 x 365 = 10 x (365/2) = 10 x 182.5 = 1825

Multiplying by 9 and 11:
E.g.: 9 * 365 = 10 x 365 – 365 = 3650 – 365 = 3285
E.g.: 11 * 365 = 10 x 365 + 365 = 3650 + 365 =4015

Difference of squares:
$a^2 - b^2 = (a + b)(a - b)$
E.g.: $51^2 - 49^2 = (51 + 49)(51 - 49) = 100 \times 2 = 200$

Simplifying squares:
E.g: $61^2 = (60 + 1)^2 = 60^2 + 2 \times 60 \times 1 + 1^2 = 3600 + 120 + 1 = 3721$

Division by 3
A number is divisible by 3 if the sum of its digits is divisible by 3.
E.g: 2328. Sum of digits = 15. 15/3 = 5, a whole number. So: 2328 is divisible by 3.

Prime factors:
A prime number is an integer only divisible by itself and 1.
Examples of primes are 2, 3, 5, 7, 11.
An integer greater than 1 can be expressed as a product of primes.
E.g.: 12 = 2 x 2 x 3.
So the prime factors of 12 are 2, 2 and 3.

Integer square roots:
Let $a = \sqrt{b}$.
Let n be the number of digits in b.
Then the number of digits in a is (n/2) rounded up to the next integer.
Look at b to estimate a range of values for a.
Use the following table to choose the last digit of a.

Last b digit	last a digit
0	0
1	1 or 9
4	2 or 8
5	5
6	4 or 6
9	3 or 7

Example:

Find the square root of 1521.
N = 4, so the number of digits in a is 4/2 = 2, rounded up if necessary.
a must be between 30 and 40 as $30^2 = 900$, and $40^2 = 1600$.
The last digit of a is 1 or 9, but as 1521 is much closer to 1600 than 900, it must be 9.
So the answer is 39.

Integer cube roots:
Let a be the cube root of b.
Let n be the number of digits in b.
Then the number of digits in a is (n/3) rounded up to the next integer.
Look at b to estimate a range of values for a.
Use the following table to choose the last digit of a.

Last b digit	last a digit
0	0
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

Example:
Find the square root of 39304.
N = 5, so the number of digits in a is 5/3 = 2 after rounding up.
a must be between 30 and 40 as $30^3 = 27000$, and $40^3 = 64000$.
The last digit of a is 4, so the answer is 34.

Number systems:

Basics:
Whole numbers W = 0, 1, 2, 3 etc
Natural numbers N = positive definite integers = 1, 2, 3 etc
Integers I or Z: . . . -2, -1 , 0, 1, 2, 3 etc
Rational numbers: expressible as a fraction J/K where J and K are integers and K is not 0.
Here J is the numerator, K is the denominator.
For an improper fraction J > K.
Surd = irrational number: not a rational number. e.g. √2, e, π.
Real numbers R: the union of the rational and irrational numbers; continuously variable numbers. E.g. 1, 1.1, √2.
Prime numbers: positive integers greater than 1 and only divisible by themselves and 1. E.g. 2, 3, 5, 7, 11 etc.

10.4567 is an example of a number in standard form for decimals. So are: 1.5, 0.678 .
10.4567 to 3 significant figures is 10.5
10.4567 to 2 decimal places (rounded to nearest last figure by default) is 10.46
10.4567 to 2 decimal places (rounded up) is 10.46
10.4567 to 2 decimal places (rounded down) is 10.45
10.4567 can also be written in scientific notation or standard form as 1.04567×10^1

Reciprocal of x = x^{-1} = 1/x

Every number can be expressed as a product of prime factors. E.g. 12 = 2 x 2 x 3 .

|x| = modulus of x. E.g. |-2| = 2 .

Ratio = (integer numerator)/(integer denominator). E.g. 2/3.

LCM, HCF, LCD
The lowest common multiple LCM of a set of numbers is the lowest number that can be divided by each. E.g. LCM of 6 and 10 is 30.

The highest common factor HCF or greatest common factor GCF of a set of numbers is the highest number which each will divide into each. E.g. HCF or GCF of 6 and 10 is 2.

The lowest common denominator LCD of a set of fractions is the lowest denominator that can be used for each fraction. E.g. LCD of ½ and 1/3 is 6, as ½ + 1/3 = 3/6 + 2/6.

Terminating and non-terminating numbers
The decimal expansion of a rational number is either terminating or non-terminating recurring.
E.g.: terminating:
  1.5
Non-terminating recurring:
  1/7 = 0.142857142857 etc
  1/11 = 0.090909 etc
  1/3 = 0.333 etc
  1/9 = 0.1111 etc

The decimal expansion of an irrational number is non-terminating non-recurring.

Rationalisation means to remove the square root from the denominator.
e.g 4/√2 = 2 √2

e
e = Euler's number = 2.71828 etc.
e = 1 + 1/1! + 1 /2! + 1/3! + 1/4! + etc.

Order of arithmetic operations:

The modern interpretation of this is:
Brackets/parentheses
Indices/exponents
Division and Multiplication
Addition and Subtraction
If two operation have the same precedence: proceed from left to right.

This is called the BIDMAS system for: brackets, indices, division and multiplication, addition and subtraction.

Example:
$6/2(1+2) = 6/2 \times (1+2) = 6/2 \times (3) = 6/2 \times 3 = 3 \times 3 = 9$
The multiplication before the first bracket is implied.

Number bases
The default base for a number is 10. This is the denary number system.
E.g. $123 = 123_{10} = 100 + 20 + 3 = 1 \times 10^2 + 2 \times 10^1 + 3 \times 1$

123 base 4 = $123_4 = 1 \times 4^2 + 2 \times 4^1 + 3 \times 1 = 16 + 8 + 3 = 27$ base 10 = 27_{10}

Modulo numbers
n modulo m or n mod m means the remainder after dividing n by m.
E.g. 11 mod 3 = 2; 15 mod 4 = 3.
(5 x 6) mod 4 = 30 mod 4 = 2.

Series:

Arithmetic series:

Let L = last term
S_n = sum to n terms
$S_n = a + (a + d) + \ldots + (a + (n - 1) d) = (n/2) [2a + (n - 1) d] = (n/2) (a + L)$
$L = T_n$ = term n = $a + (n - 1) d$

Let $T_r = r$, then $S_n = (n (n + 1))/2$

Let $T_r = r^2$, then $S_n = (n (n + 1) (2n + 1))/6$

Let $T_r = r^3$, then $S_n = (n^2(n + 1)^2)/4$

Geometric series:

$S_n = a + ar + ar^2 + \ldots ar^{(n - 1)} = a(1 - r^n)/(1 - r)$, r not equal to 1.
$T_n = a r^{n-1}$

Infinite geometric series:

$S = a + ar + ar^2 + \ldots = a/(1 - r), |r| < 1.$

Binomial expansion:

$(x + y)^n = C_0^n x^n + C_1^n x^{n-1} y + C_2^{n-2} y^2 + \ldots + C_n^n y^n$

Example expansions:

$2 = 1 + ½ + ¼ + 1/8 + \ldots$

$e^x = 1 + x/1! + x^2/2! + x^3/3! + \ldots$

$e^{i\pi} = -1$ where $i = \sqrt{-1}$

$\pi/4 = 1 – 1/3 + 1/5 - \ldots$

$\pi^2/6 = 1 + ¼ + 1/9 + 1/16 + \ldots$

$\sin(x) = x – x^3/3! + x^5/5! - \ldots$

$\cos(x) = 1 – x^2/2! + x^4/4! - \ldots$

Quadratics

Solve $a x^2 + b x + c = 0$	$x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$
	Sum of roots = -b/a Product of roots = c/a Discriminant = $b^2 – 4ac$

Equation of a parabola: $y = ax^2 +bx +c$
Equation of a parabola symmetrical about (h, k): $(y - k) = a (x – h)^2$

Parametric representation of a parabola
For $x^2 = 4ay$, let $x = 2at$ so that $y = at^2$.
At $(2at, at^2)$:
Tangent: $y = tx – at^2$
Normal: $x + ty = at^3 + 2at$
At (x_1, y_1):
Tangent: $xx_1 = 2a (y + y_1)$
Normal: $y – y_1 = -(2a/x_1) (x - x_1)$
Chord of contact from (x_0, y_0): $xx_0 = 2a (y + y_0)$

Polynomials

A polynomial is an algebraic expression in which variables have non-negative integral powers. E.g.: $p(x) = x^2 + 2x -3$ is a second order or second degree polynomial with single variable x. It is a trinomial as it has 3 terms.
If p(a) = 0 then a is a zero of the polynomial p(x).
Finding the zeroes of p(x) means solving the equation p(x) = 0.
A monomial is a polynomial with only one term. E.g.: p(x) = 3x, or p(x, y) = 3xy.

Remainder theorem:
If polynomial p(x) has degree > 0 then the remainder of p(x)/(x - a) is p(a).

Factor theorem:
If polynomial p(x) has degree > 1 then if p(a) = 0 then (x - a) is a factor of p(x).

Cubic equation
$ax^3 + bx^2 + cx + d = 0$

Let roots be x_1, x_2, x_3.
Then:

$(x - x_1)(x - x_2)(x - x_3) = 0$
$x_1 + x_2 + x_3 = -b/a$
$x_1x_2 + x_2x_3 + x_3x_1 = c/a$
$x_1x_2x_3 = -d/a$

Alternative formula:
$y = a(x - h)^3 + k$ Inflexion is at (h, k).

Estimation of roots of a polynomial equation.

Start at x_1, then x_2 is an estimate of a root.

$x_2 = x_1 – f(x_1)/f\,`(x_1)$

Unit Conversions

Angles
The degrees of arc in a circle = 360.
180 degrees = π radians
1 degree of arc = 60 minutes
1 minute of arc = 60 seconds

Distance
1 meter = 1 metre = 1 m = 100 centimetres = 100 cm
1 inch = 2.54 cm
1 foot = 12 inches
1 yard = 3 feet
1 mile = 1760 yards
1 nautical mile = 1 minute of latitude = 1852 metres

Area
1 hectare = 1 ha = 10000 m^2
1 acre = 4840 square yards

Volume
1 m^3 = 1000L
1L = 1 litre = 1000cc = 1000 cubic centimetres
1 pint = 0.56826 litres
1 quart = 2 pints
1 gallon = 4 quarts

Information
1 bit = 0 or 1
1 byte = 8 bits
1 kilobyte = 2^{10} bytes = 1024 bytes = 1KB
1 megabyte = 2^{20} bytes = 1024 kilobytes = 1MB
1 gigabyte = 2^{30} bytes = 1024 megabytes = 1GB
1 terabyte = 2^{40} bytes = 1024 gigabytes = 1TB

Prefixes

p = pico = 10^{-12}
n = nano = 10^{-9}
μ = micro = 10^{-6}
m = milli = 10^{-3}
centi = 10^{-2}
deci = 10^{-1}
deca = 10
k = kilo = 10^{3} = 1 thousand
M = mega = 10^{6} = 1 million
G = giga = 10^{9} =1 billion

Weight
t = tonne = 10^{3} kg
16 ounces = 1 pound = 1 lb = 0.4536 kg
1 ton = 2240 lb

Temperature:
A^{o} C = (B^{o} F -32) x (100/180)

Complex Numbers

Let $i = \sqrt{-1}$, then $i^2 = -1$, and $z = a + ib$ is a complex number.
C is the set of complex numbers. So $z \in C$.
Re(z) = real part of z.
Im(z) = imaginary part of z.

Note that *i* is called an imaginary number. It is an artificial construct that has been found to very useful in many areas of science.

$z = a + ib = r\ cis(\theta) = r(\cos(\theta) + i \sin(\theta))$
Note that anticlockwise is positive for θ.

$-z = -a - ib$

$z^2 = (a + ib)^2 = a^2 + 2abi - b^2$

$|z| = \sqrt{(a^2 + b^2)}$

Complex conjugate of $z = z^* = a - ib = r\ cis(-\theta) = r(\cos(\theta) - i \sin(\theta))$

$|z|^2 = z z^* = (a + ib)(a - ib) = a^2 + b^2$

$1/z = 1/(a + ib) = (a - ib)/(a^2 + b^2)$

$e^{i\pi} = -1$

$e^{i\theta} = \cos(\theta) + i \sin(\theta) = cis(\theta)$

If $z = |z| e^{i\theta}$, then $Arg(z) = \theta$
$-\pi < Arg(z) <= \pi$

de Moivre's theorem:

$e^{in\theta} = (\cos(\theta) + i \sin(\theta))^n = \cos(n\,\theta) + i \sin(n\,\theta) = (\text{cis}(\theta))^n = \text{cis}(n\,\theta)$

Polar co-ordinates:
$z = r\,e^{i\theta}$

$z_1 z_2 = r_1 r_2 \text{ cis}(\theta_1 + \theta_2)$

Complex roots example:
Find the roots of $x^2 + x + 1 = 0$
The formula gives:
$x = (-1 \pm \sqrt{(-3)}) / 2 = (-1 \pm \sqrt{3}\, i) / 2$

Argand plane
An *Argand diagram* is a plot of *complex* numbers as points in the *complex plane* using the x-axis as the real axis and y-axis as the imaginary axis.
The unit in the y-axis direction is i.
Point (3, 4) represents complex number 3 + 4i .
For example: Im(z) can be plotted against Re(z).